GCSE Maths – Algebra

Simultaneous Equations

Notes

WORKSHEET

Simultaneous Equations

Linear simultaneous equations

To solve two linear simultaneous equations, we use **elimination**. One of the variables is eliminated by subtracting one equation from the other.

Example: *Solve the following simultaneous equations by elimination*
$$2x + 4y = 26$$
$$3x + y = 4$$

1. Choose which **variable** you will **eliminate**. Make sure the **coefficient** of that variable is the **same** for each equation.

   To eliminate the y variable, multiply the second equation by 4:

   $$2x + 4y = 26$$
   $$12x + 4y = 16$$

2. **Add** or **subtract** the equations to eliminate the required variable. In our case, subtract the second equation from the first:
   $$2x + 4y = 26$$
   $$(-) \quad 12x + 4y = 16$$
   $$\Rightarrow -10x = 10$$
   $$\Rightarrow x = -1$$

3. **Substitute** the value found into one of the original equations to find the value of the other variable.
   $$2(-1) + 4y = 26$$
   $$-2 + 4y = 26$$
   $$4y = 28$$
   $$y = 7$$
   So, the final answer is $x = -1$, $y = 7$.

Find approximate solutions using a graph

The point of intersection of two lines on a graph gives the solution to the simultaneous equations of the lines.

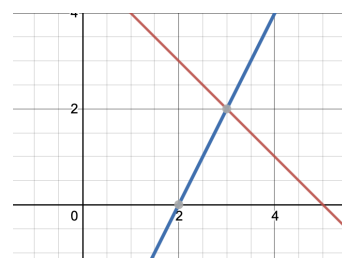**Example**: *Using the graph below, find the solution to the simultaneous equations*
$$y = -x + 5$$
$$y = 2x + 4$$

Here, the first line $y = -x + 5$ is plotted by using the fact that its y-intercept is 5 and its gradient is -1.

The second line $y = 2x + 4$ is plotted by using the fact that its y-intercept is -4 and its gradient is 2.

The lines intersect at the point $(3,2)$ so the solution is $x = 3, y = 2$.

Non-linear simultaneous equations (Higher only)

Non-linear simultaneous equations appear when one of the equations is quadratic and the other is linear. To solve non-linear simultaneous equations, we use **substitution**. This is where the linear equation is substituted into the quadratic equation.

> **Example**: *Solve the following simultaneous equations by substitution*
> $$y + x = 4 \text{ (linear)}$$
> $$y = x^2 + 3x - 1 \text{ (quadratic)}$$
>
> 1. Rearrange the linear equation to get one of the unknowns on its own.
> $$y = 4 - x$$
>
> 2. **Substitute** the linear equation into the quadratic equation and then **solve** the quadratic equation.
> $$4 - x = x^2 + 3x - 1$$
> $$x^2 + 4x - 5 = 0$$
> $$(x + 5)(x - 1) = 0$$
> $$x = -5 \text{ or } x = 1$$
>
> 3. **Substitute** each value found into the original linear equation to obtain the corresponding value of the other variable.
> $$\text{When } x = -5, y = 4 - (-5) = 9$$
> $$\text{When } x = 1, y = 4 - (1) = 3$$
>
> *So, the solutions to the simultaneous equations are $x = -5, y = 9$ and $x = 1, y = 3$.*

Find approximate solutions using a graph (Higher only)

Similar to linear simultaneous equations, the approximate solutions of non-linear simultaneous equations can be found by taking the **points of intersection** of the curves of the equations.

> **Example:** *Using the graph, find the solution to the simultaneous equations*
> $$y = x - 2$$
> $$y = x^2 + x - 6$$
>
> *The first line $y = x - 2$ is plotted by using the fact that its y-intercept is -2 and its gradient is -2.*
> *The quadratic curve $y = x^2 + x - 6$ is plotted by finding where the curve crosses the axes:*
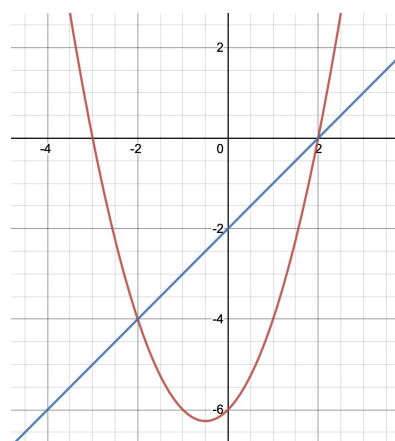>
> *When $x = 0$: $y = -6 \Rightarrow (0, -6)$*
> *When $y = 0$: $x^2 + x - 6 = 0$*
> $$(x + 3)(x - 2) = 0$$
> $$x = -3 \text{ or } x = 2$$
> $$\Rightarrow (-3, 0) \text{ and } (2, 0)$$
>
> *The curves intersect at the points $(2, 0)$ and $(-2, -4)$. So, the solutions are $x = 2, y = 0$ and $x = -2, y = -4$.*
>

Simultaneous equations – Practice Questions

1. Solve the simultaneous equations
    $$7a + 3b = 51$$
    $$4a + b = 22$$

2. Solve the simultaneous equations
    $$5c - 4d = 29$$
    $$c + 3d = 21$$

3. By method of sketching a graph, solve the simultaneous equations

    $$y = 2x + 3$$
    $$y = -x + 6$$

4. (Higher only) Solve the simultaneous equations

    $$y + x = 3$$
    $$y = x^2 - 2x + 1$$

5. (Higher only) Solve the simultaneous equations

    $$2x + 4y = 14$$
    $$x^2 + 3y - x = 27$$

6. (Higher only) By method of sketching a graph, find approximate solutions to the simultaneous equations

    $$y = (x - 4)^2 + 3$$
    $$y = 2x - 2$$

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.